Homework 2 solution

February 24, 2016

Problem 1. Digital audio is sampled at a rate of 44,100 samples per second. What is the range of frequencies that can be captured?

Solution:

$$\text{sampling rate} = 2 \times f_{\max} \tag{1}$$

$$f_{\max} = \frac{44100}{2} = 22050\ Hz \tag{2}$$

So the range of frequencies that can be captures is between 0 and 22050.

Problem 2. If a microphone signal is sampled at $44,100$ samples per second and each sample is 16 bits, what is the rate at which digital data is produced?

Solution:

$$\text{data rate} = 44100\frac{\text{samples}}{\text{second}} \times 16\frac{\text{bits}}{\text{sample}} = 705600\frac{\text{bits}}{\text{second}} \tag{3}$$

Problem 3. Suppose a ship uses colored lights to communicate with another ship, and there are six possible colors (red, blue, yellow, green, purple, and white). Further suppose that white is reserved to start and end each message, and that once a message starts one of the other color lights is flashed once every two seconds. If the lights are used to transmit digital information, what is the data rate during message transmission (i.e., during the time between the white lights)? Suppose the purple flag is accidentally lost (e.g., dropped overboard). Does your answer change? Why?

Solution:

Equation 4 expresses the relationship between baud, signal levels, and bit rate:

$$\text{bits per second} = \text{baud} \times \left\lfloor \log_2^{levels} \right\rfloor \tag{4}$$

Part a

The baud rate is the number of times a signal can change per second. Because one of the color lights is flashed every two seconds, we have 0.5 flashes per second. Consequently, the baud rate is 0.5 flashes/second. In addition, there are 5 levels because we have 5 possible colors. So, we can compute the bit rate as follows:

$$0.5 \times \lfloor \log_2^5 \rfloor = 1 \frac{\text{bits}}{\text{second}} \tag{5}$$

Part b

When the purple light is lost then the number of levels will be reduced to 4 so we will have:

$$0.5 \times \lfloor \log_2^4 \rfloor = 1 \frac{\text{bits}}{\text{second}} \tag{6}$$

As we can see from the above result, the answer is not changed.

Problem 4. If an optical fiber has a bandwidth of 2 Gigahertz and a modem uses 512 signal levels, what is the maximum data rate according to Nyquist?

Solution:

Theoretical bound on maximum data rate for bandwidth B and k signal levels can be computed by using equation 7:

$$D = 2 \times B \times \log_2^k \tag{7}$$

So, the maximum data rate for the optical fiber that is explained above is:

$$2 \times 2 \times 10^9 \times \log_2^{512} = 36 \times 10^9 \frac{\text{bits}}{\text{second}} = 36 \frac{\text{Gbits}}{\text{second}} \tag{8}$$

Problem 5. Using the fiber in the previous question, if the average signal power is 405 units and the average noise power is 27 units, what is the maximum channel capacity according to Shannon?

Solution:

Based on Shannon's theorem, maximum channel capacity in the presence of noise can be computed by using equation 9:

$$C = B \times \log_2^{(1+\frac{S}{N})} \tag{9}$$

Consequently, the maximum channel capacity is:

$$2 \times 10^9 \times \log_2^{(1+\frac{405}{27})} = 8 \times 10^9 \frac{\text{bits}}{\text{second}} = 8 \frac{\text{Gbits}}{\text{second}} \tag{10}$$